# Classical ADE and PACO Omnidirectional Dual-Reflector Antennas Simulated in 2-D Using a Nystrom-Type MDS Algorithm

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***Abstract*— A full-wave numerical study of 2-D models of dual-reflector omnidirectional ADE (axially-displaced ellipse subreflector and parabolic main reflector) and PACO (parabolic subreflector and corner reflector) antennas is performed in the E-polarization case. The analysis is based on the recently developed Method of Discrete Singularities (MDS), which uses coupled electric-field singular integral equations (SIE) applied to accurate modelling of the considered dual-reflector (in fact, three-reflector in the considered 2-D case) antenna systems.**

## I. Introduction

Axially symmetric designs of PACO (Fig. 1a) and ADE (Fig. 1b) omnidirectional antennas, together with a horn feed, are aimed at providing omnidirectional coverage with a horizontally directed or slightly tilted main beam, suited for the use in the base stations of point-to-multi-point radio, TV and internet broadband links. This configuration is based on quasioptical considerations and usually is modelled with Geometrical Optics (GO) applied to cross-sectional 2-D geometry [1-4] or with MoM [5,6].

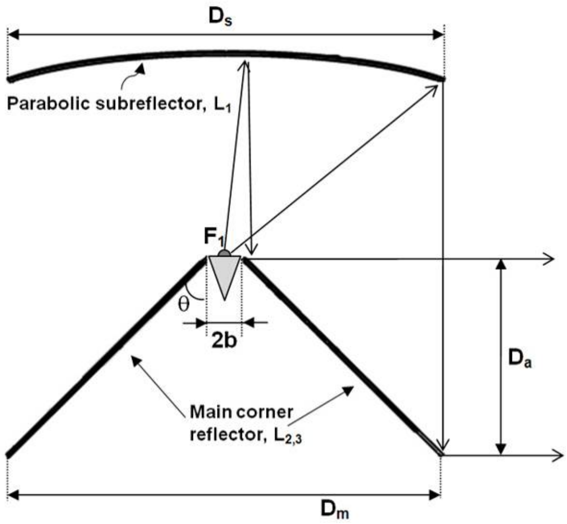


Fig. 1a. Parabola-corner (PACO) model of omnidirectional antenna.

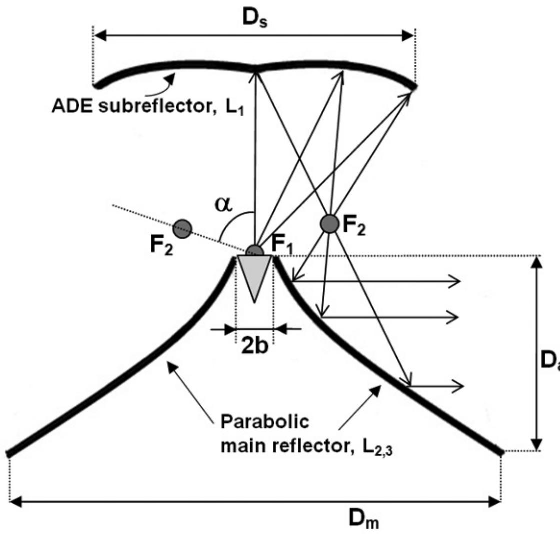


Fig. 1b. Axially-displaced ellipse (ADE) model of omnidirectional antenna.

As GO is a high-frequency approximation, realistic antenna modelling can be greatly improved by using a full-wave method of analysis. Here, due to the large electrical size of reflectors, application of both conventional MoM simulations and FDTD require huge computer resources. As a remedy, we use MDS to discretize coupled SIEs into economic-size matrix equations possessing fast convergence and controlled accuracy. MDS is in fact a Nystrom-type method of solving the SIEs in wave scattering by flat strips; it is similar to [7], but uses different interpolation formulas.

## II. MDS Analysis

### *A. Problem Formulation*

The geometry of generic 2-D three-element structures viewed as cross sections of an omnidirectional antenna can be seen in Figs. 1a and 1b. The reflectors $L_q$ ($q$=1,2,3) are assumed to be perfect electric conductors (PEC) and have zero thickness. The feed is a line current placed at the complex-valued source point (CSP) [9] and has harmonic time dependence omitted in the analysis. The field generated by such a feed can be characterized by the $z$-component of the electric field which is given by the Hankel function of a complex argument. This function simulates a Gaussian beam in the near zone of paraxial domain and smoothly transforms into cylindrical wave off this domain. It has two branch points, which lead to a cut in the real space that can be considered as a model of the real horn aperture with a length of $2b$. The feed directivity is controlled by parameter $kb$ ($k$ is the free-space wavenumber): the greater $kb$, the narrower the beam.

*B. MDS: System of SIEs and Discretization*

The total field here is a sum of the field scattered by all reflectors and the incident field: $U = U_{sc}+U_0$. The function $U_{sc}$ has to solve the Helmholtz equation off $L_q$ and satisfy three conditions: (a) the PEC boundary condition on $L_q$, (b) the edge conditions at the end points, and (c) the radiation condition. In E-polarization, this problem is reduced to a set of 3 coupled SIEs for the surface currents induced on reflectors, $j_q(s)$:

$$\frac{i}{4}\sum_{q=1}^{3}\int_{L_q} H_0^{(1)}\left(k\left|\vec{r}_q(s)-\vec{r}_p(s_0)\right|\right) j_q(s)ds = -U_0\left(\vec{r}_p(s_0)\right)\Big|_{L_p} \quad (1)$$
$$p = 1,2,3$$

Note that the integrals involved in Eqn. (1) have logarithmic singularities in their kernels. Therefore, their discretization requires an efficient and convergent numerical algorithm.

Applying a contour parameterization to $L_q$, one can transform SIEs (1) to another set of Cauchy-singular SIEs with smooth supplementary conditions and with new unknown current functions. This new set of SIEs is further discretized by using the quadrature formulas of interpolation type with the nodes in the nulls of the Chebyshev polynomials of the first and second kind. As a result, in the multi-reflector case we obtain coupled sets of algebraic equations, the solution of which gives us the sought surface current functions.

The mathematical details of this method of numerical solution of SIEs can be found in [8]-[11]. It enables one to study the effects of the wave radiation, guidance and scattering in reflector antennas and waveguides with high accuracy using desktop computer resources. Surface currents, near- and far-field patterns, radiated and scattered power and gain behaviour can be readily computed for various reflector shapes, feed locations, etc.

## III. Numerical Results

We consider two different design concepts in the case of E-polarization. The classic omnidirectional axially-symmetric configuration consists of a parabolic subreflector and a conical main reflector fed by a coaxial horn (Fig. 1a). An alternative dual-reflector design is based on ADE, first studied in [3], where the subreflector is an ellipsoid, with one of the foci displaced from the symmetry axis and coinciding with the focus of parabola, which forms the main reflector (Fig. 1b). In this arrangement, due to the inversion of the feed radiation provided by the ADE subreflector, the feed main-beam radiation can be directed away from the feed aperture, thus reducing the overall return loss. After rotation around the symmetry axis, this focus follows a caustic ring between the sub-reflector and the main reflector. The dual-reflector system provides collimation of the spherical wavefront radiating from the system focus into a cylindrical wavefront at the main reflector aperture. Both antennas were designed to fit into the same dimensions (30λ in width × 45λ in height). The gap holding the feed aperture had the length of (2$b$+0.2λ). The analysis involved plotting directivities (Fig. 2) as functions of $kb$ to determine the best illumination of the PACO and ADE subreflectors, and building the near-fields (Figs. 3a, 4a), phase patterns (Figs. 3b, 4b) and the far-fields (Figs. 7a, 7b) of the considered PACO and ADE 2-D models of omnidirectional antenna designs. The geometrical parameters used in simulations are given in Table 1.

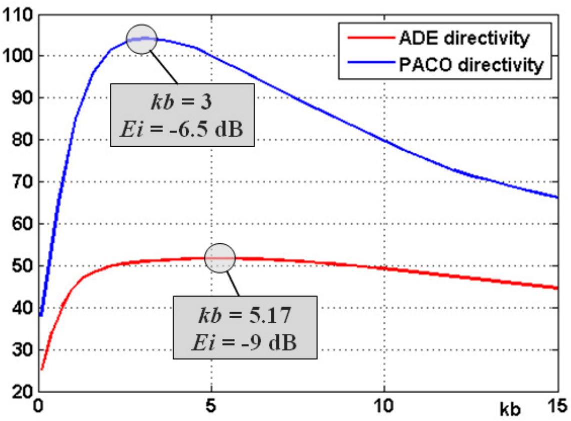


Fig. 2. Directivities of both antennas as a function of $kb$. Optimal $kb$ values and corresponding edge illuminations of the subreflectors are shown.

The main conclusion of this comparative study is that the PACO antenna has twice larger directivity value than the ADE antenna of the same cross-sectional size.

TABLE I
GEOMETRICAL PARAMETERS USED IN SIMULATIONS OF PACO AND ADE ANTENNAS

| | PACO | ADE |
|---|---|---|
| $D_s$ | 30λ | 25.8λ |
| $D_m$ | 34λ | 30λ |
| $D_a$ | 16λ | 14λ |
| θ | $45^0$ | -- |
| α | -- | $11.5^0$ |
| $kb$ | 3 | 5.17 |
| $L_{1,2,3}$ | [30.6,22,22]λ | [26.7,19,19]λ |

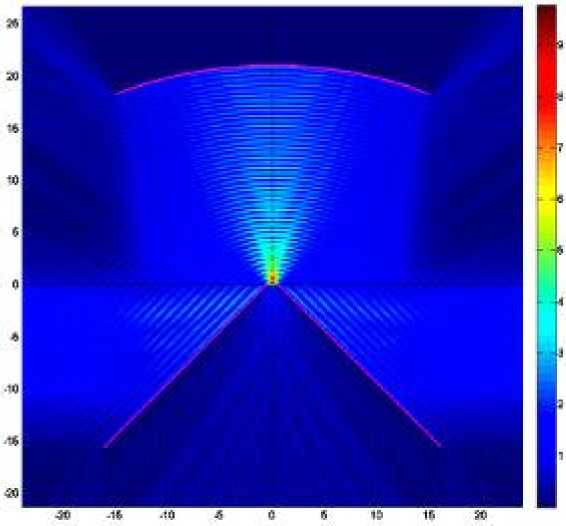

Fig. 3a. Near-field map of the PACO antenna.

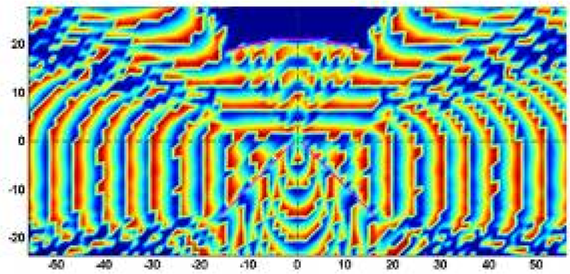

Fig. 3b. Near-field phase pattern for the PACO antenna considered in Fig. 3a.

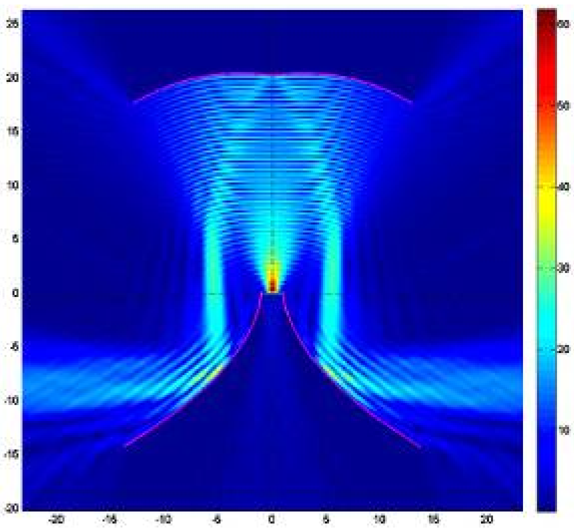

Fig. 4a. Near-field map of the ADE antenna.

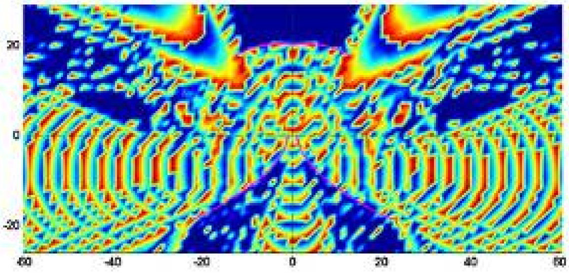

Fig. 4b. Near-field phase pattern for the ADE antenna considered in Fig. 4a.

However, it appears possible to increase significantly the ADE antenna directivity by designing the parabolic main reflectors to be shallower. This modification shifts the fragments of the parabolic reflectors lower of their original position in Fig. 4a and increases the vertical dimensions of the antenna system to 55λ (in comparison to 35λ in Fig. 4a), but shapes the wavefront closer to cylindrical one. The other geometrical dimensions remain unchanged. Fig. 5 shows the directivity plot of the modified ADE antenna in comparison to the directivities of PACO and original ADE antennas studied above. The maximum directivity of the modified ADE antenna almost reaches the maximum of the PACO antenna (96 versus. 104 a.u.). The optimal *kb* value of the modified optimized ADE antenna is *kb* = 8.

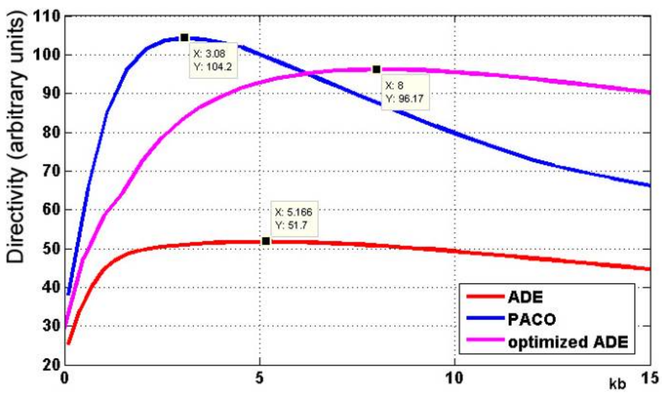


Fig. 5. Directivities of PACO, ADE and modified ADE antennas as functions of *kb*. Optimal *kb* values and corresponding directivity values in arbitrary units are shown.

Note that the feed in this case is no longer positioned in the gap between the parabolic mirrors but elevated at some distance over it (see Fig. 7a). Therefore realization of such design will need elongated metallic pole along the axis of the antenna that contains a feeding waveguide. This configuration may need additional modelling and optimization for best performance and reduced back-scattering. Fig. 6a shows the near-field for a modified optimized ADE antenna and its phase pattern is shown in Fig. 6b. Fig. 7a shows the far-field comparison for all three 2-D models of the studied omnidirectional antenna types.

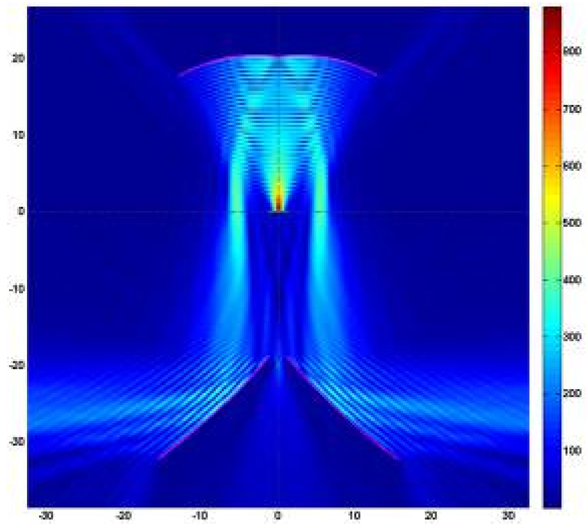

Fig. 6a. Near-field map of the modified optimized ADE antenna.

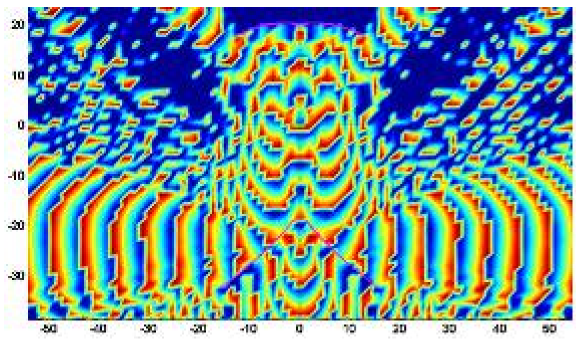

Fig. 6b. Near-field phase pattern for the ADE modified optimized antenna considered in Fig. 7a.

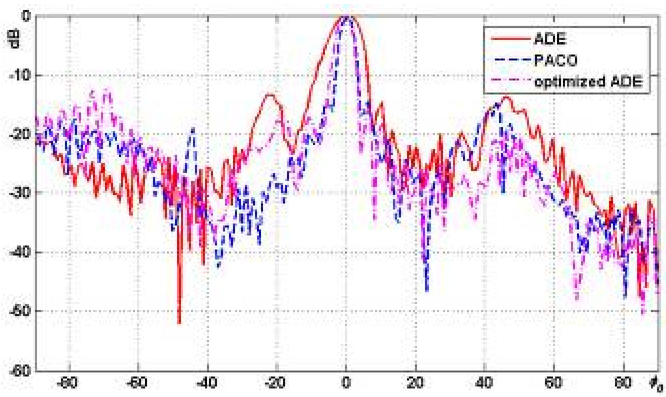


Fig. 7a. Log radiation pattern comparison for ADE, PACO and modified optimized ADE antennas.

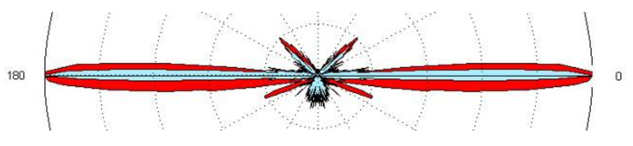

Fig. 7b. Polar radiation pattern comparison for ADE (red, outer) and PACO (blue, inner) antennas

## IV. CONCLUSIONS

We have presented a 2-D albeit accurate and efficient numerical analysis of two different designs of quasi-optical omnidirectional dual-reflector antennas. Such antennas are used for point-to-multi-point wireless transmission links. The analysis has been done with the SIE-MDS approach that we had earlier applied to the multireflector antennas and beam waveguides. A performance comparison of ADE and PACO omnidirectional designs revealed the PACO being more directive than the ADE antenna due to the shallow parabola fragment used in the latter case, chosen to reduce the dimensions of the ADE system. The ADE directivity can be drastically increased by introducing a shallower parabola segments in the antenna design.

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